R-regression in heteroscedastic or nonlinear models

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At first we study a simple regression model

$$Y_i = \beta_0 x_i + \epsilon_i\,, \;\; i = 1, \ldots, n\,,$$

where the errors ϵ_i are independent but non-identically distributed. The error distributions depend on the regression constants $x_1, \ldots, x_n$ and are symmetric. We derive a local stochastic expansion of the linear rank statistic

$$S_n(\beta) = \sum_{i=1}^{n} x_i a_n[R(Y_i - \beta x_i)]\,,$$

where $R(Y_i - \beta x_i)$ denotes the rank of $Y_i - \beta x_i$. We study under which conditions is the R-estimate of β_0 a consistent estimate and show that the estimate is asymptotically normally distributed.

Furthermore we consider a nonlinear regression model

$$Y_i = g(x_i, \beta_0) + \epsilon_i\,, \;\; i = 1, \ldots, n\,,$$

with i.i.d. errors $\epsilon_1, \ldots, \epsilon_n$. We derive an analogous stochastic expansion for the related nonlinear rank statistic

$$S_n(\beta) = \sum_{i=1}^{n} g'(x_i, \beta) a_n[R(Y_i - g(x_i, \beta))]$$

and discuss the consistency of the nonlinear R-estimate.