Bouligand Derivatives and Robustness of Support Vector Machines

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We investigate robustness properties for a broad class of support vector machines with non-smooth loss functions. These kernel methods are inspired by convex risk minimization in infinite dimensional Hilbert spaces. Leading examples are the support vector machine based on the ϵ-insensitive loss function, and kernel based quantile regression based on the pinball loss function. Firstly, we propose with the Bouligand influence function (BIF) a modification of F.R. Hampel's influence function. The BIF has the advantage of being positive homogeneous which is in general not true for Gâteaux-derivatives like the influence function. Secondly, we show that many support vector machines based on a Lipschitz continuous loss function and a bounded kernel have a bounded Bouligand influence function and are thus robust.

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