Fine properties of the Pitman estimators in small samples

Abram Kagan

Dept. of Mathematics, University of Maryland, College Park, MD 20742, USA

Abstract

Let $t_n = t_n(x_1, \ldots, x_n)$ be the Pitman estimator (with respect to the quadratic loss function) of the parameter θ from a sample $(x_1, \ldots, x_n)$ from a population $F(x-\theta)$. Assuming only that $\int x^2 dF(x) < \infty$ (even absolute continuity of F is not required), we prove some properties of $\mathrm{var}(t_n)$. In particular, for any $n \geq 1$

$$n\mathrm{var}(t_n) \geq (n+1)\mathrm{var}(t_{n+1})$$

(it seems likely that the equality sign holds only for Gaussian F). The case when $\lim_{n\to\infty} n\mathrm{var}(t_n) > 0$ (the limit always exists) should be called regular and it is an open problem to find out if regularity implies the finiteness of the Fisher information in F. Some related results will be discussed (it is a joint work with Tinghui Yu).