Translation invariant and positive homogeneous risk measures and portfolio management.

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Abstract

Translation invariant and positive homogeneous risk measures are of the significant interest for solving noteworthy problems of financial economics and actuarial risk management. This class consists of many important measures such as Value-at-risk (VaR), Expected Short Fall (ES) and Standard Deviation Premium (STD) among others; ES has other names: Tail Condition Expectation (TCE), Tail VaR, Conditional VaR (CVaR). We show that any risk measure of the portfolio of risks from the considered class of measures reduces to the combination of linear and square root functionals of the coefficients of the portfolio diversification in the case of multivariate elliptical distributed risks (the multivariate normal distribution is an important example). We give the condition when the problem of minimization of this combined functional under the system of linear constraints has the finite solution and provide the explicit closed form solution. For the case when the mean of the portfolio is certain this solution is equivalent to a well-known mean-variance Markowitz portfolio. If the latter restriction is not included in the system of linear constraints the solution is distinct from that obtained by mean-variance risk measure. As a main corollary we solve the problem of minimizing the VaR and ES risk measures. The results are demonstrated with the data of stocks from NASDAQ/Computers.